MATH 1300, Mathematical Explorations

Open Questions

Activity

- Have students complete one or more of the following activities - assign different activities to different tables/groups

  1. The Collatz graph has vertices $1, 2, 3, \ldots$ It has a directed edge (an arrow) from n to $n/2$ for every even number n, and has a directed edge from n to $3n+1$ for every odd number n. Draw as much of the Collatz graph as possible.
  2. Draw some billiard paths in equilateral, 30-60-90, and 45-45-90 triangles. Search for periodic paths.
  3. Suppose we have a hexagonal tessellation of the plane by hexagons of diameter a little less than 1 (say 0.9 if you like). Show that given 7 colors, you can assign colors to the hexagons so as to color the plane is such a way that no pair of points a distance exactly 1 apart have the same color.
  4. Show that if we are going to assign colors to the given graph in such a way that no vertices connected by an edge have the same vertex, then we will need at least four colors.
  5. Write down some even numbers that are greater than 2 (large ones if you are feeling ambitious) and express them as the sum of two primes.
  6. Draw some loops and find the vertices of a square on them. (E.g. ellipses, triangles,....)
  7. Construct several rows of Pascal's triangle. Find some patterns. Find some numbers (other than 1) that occur frequently in the triangle. How many times do they occur?
  8. The divisors of 10 are 1, 2 and 5. (We exclude 10 itself from our list of divisors.) Their sum is $1 + 2 + 5 = 8$. Sum the divisors of 6, 28, 496, 8128, and 33550336.

- Talk about examples of open questions (pick questions which go along with any of the above activities that student's worked on):

  1. The Collatz Conjecture. Has been checked for all numbers up to 5×10^{18}. (See Conway's On Unsettleable Arithmetical Problems.)
  2. Goldbach's conjecture: every even integer greater than 2 can be expressed as the sum of two primes.
  3. What is the chromatic number of the plane. (It is known to be between 4 and 7.)
  4. Perfect numbers: $6, 28, 496, 8128, 33550336, \ldots$ Are there infinitely many? Are any of them odd?
  5. A point travels with unit speed inside the triangle, bouncing off the sides according to the usual rule from geometric optics: angle of incidence equals angle of reflection. Does there always exist a trajectory which will be periodic?loop back on itself?

6. Does every simple closed curve in the plane contain all four vertices of some square?
7. Singmaster's conjecture. Does any number (apart from 1) appear in Pascal's triangle more than 8 times? (3003 appears 8 times.)
8. Is $e + \pi$ rational?
9. Twin prime conjecture.

- Gallery walk: put questions on the board and have students walk around and write answers.
  - What are some words you would use to describe mathematics?
  - In what respect has your perception of math changed the most this semester?
  - What activities did you like the most?
  - What activities did you not like?
  - Do you have any suggestions for next time we teach this?
  - What would you like to know more about?

Notes

This is a good end of semester activity/last day of class activity.

Assignments

1. There have been many famous theorems in mathematics that remained unsolved for decades (and, sometimes, centuries). The most famous of these is Fermat's Last Theorem. Watch "The Mathematician and the Devil." This movie is from the 70's (and has many problematic aspects) but it is the best portrayal of a mathematician that I have seen in film. Write a response to the movie, as well as the idea of extremely old problems (such as Fermat's Last Theorem, the Four Color Theorem, and the Collatz conjecture). Does the idea of solving an old problem intrigue you? inspire you? intimidate you? Do you ever see these kinds of motivations appearing in your own life?

References and resources

AMS Blog: Unsolved Problems in Math

Movie: The Mathematician and the Devil